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Production Inventory Model with Different Deterioration Rates Under Linear Demand

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Abstract:- A production inventory model with linear trend in demand is developed. Different deterioration rates are considered in a cycle. Shortages not allowed. To illustrate the model numerical example is provided and sensitivity analysis is also carried out for parameters.

Key Words: *Production, Inventory model, Varying Deterioration, Linear demand, Time varying holding cost*

I. INTRODUCTION

The basic question for making a production inventory model is how much to produce to satisfy customer demand. So, the problem is to find the optimal time to maximize the total relevant profit. Moreover, deterioration of certain items occurs due to spoilage, obsolescence, evaporation, etc. Many authors in past studied inventory model for deteriorating items. Covert and Philip [2] derived an EOQ model for items with weibull distribution deterioration. Mishra [7] developed the EPQ model for deteriorating items with varying and constant rate of deterioration. Aggarwal [1] discussed an order level inventory model with constant rate of deterioration. Gupta and Vrat [4] developed an inventory model with stock dependent consumption rate. Mandal and Phaujdar [6] presented an inventory model for stock dependent consumption rate. Haiping and Wang [5] studied an economic policy model for deteriorating items with time proportional demand. Other research work related to deteriorating items can be found in, for instance (Raafat [9], Goyal and Giri [3], Ruxian et al. [11]).

Roy and Chaudhary [10] studied a production inventory model for deteriorating items when demand rate is inventory level dependent and production rate depends both on stock level and demand. Tripathy and Mishra [12] developed an order level inventory model with time dependent Weibull deterioration and ramp type demand rate where production and demand were time dependent. Patel and Patel [8] developed a deteriorating items production inventory model with demand dependent production rate.

Initially there is no deterioration in most of the products. After certain time deterioration starts and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.

In this paper we have developed a production inventory model with different deterioration rates for the cycle time under time varying holding cost. Shortages are not allowed. Numerical example is provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is also carried out.

II. ASSUMPTIONS AND NOTATIONS

Notations:

The following notations are used for the development of the model:

P(t) : Production rate is a function of demand ($P(t) = \eta D(t)$, $\eta>0$)

D(t) : Demand rate is a linear function of time t ($a+bt$, $a>0$, $0<b<1$)

A : Replenishment cost per order

c : Purchasing cost per unit

p : Selling price per unit

T : Length of inventory cycle

I(t) : Inventory level at any instant of time t, $0 \le t \le T$

Q : Order quantity

θ : Deterioration rate during $\mu_1 \le t \le t_1$, $0< \theta<1$

θt : Deterioration rate during , $t_1 \le t \le T$, $0< \theta<1$

π : Total relevant profit per unit time.

Assumptions:

The following assumptions are considered for the development of the model:

- The demand of the product is declining as a linear function of time.
- Rate of production is a function of demand

- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are not allowed.
- Deteriorated units neither be repaired nor replaced during the cycle time.

III. THE MATHEMATICAL MODEL AND ANALYSIS

Let I(t) be the inventory at time t ($0 \leq t \leq T$) as shown in figure.

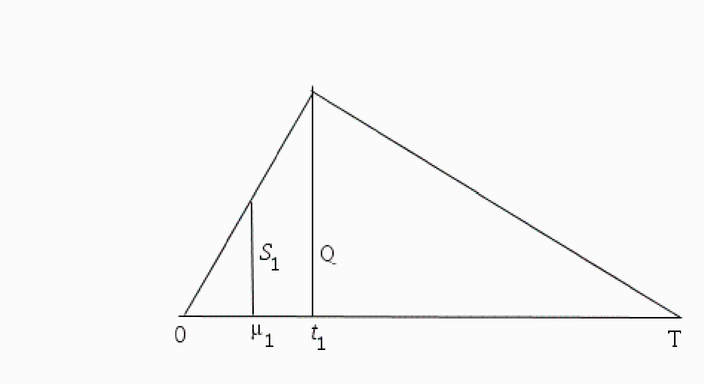


Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by

$$\frac{dI(t)}{dt} = (\eta - 1)(a + bt), \qquad 0 \leq t \leq \mu_1 \qquad (1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = (\eta - 1)(a + bt), \qquad \mu_1 \leq t \leq t_1 \qquad (2)$$

$$\frac{dI(t)}{dt} + \theta t I(t) = -(a + bt), \qquad t_1 \leq t \leq T \qquad (3)$$

with initial conditions I(0) = 0, $I(\mu_1) = S_1$, $I(t_1) = Q$ and I(T) = 0.

Solutions of these equations are given by

$$I(t) = (\eta - 1)(at + \frac{1}{2}bt^2). \qquad (4)$$

$$I(t) = S_1\left[1+\theta(\mu_1 - t)\right] - (\eta-1)\left[a(\mu_1 - t) + \frac{1}{2}b(\mu_1^2 - t^2) + \frac{1}{2}a\theta(\mu_1^2 - t^2) + \frac{1}{3}b\theta(\mu_1^3 - t^3) - a\theta t(\mu_1 - t) - \frac{1}{2}b\theta t(\mu_1^2 - t^2)\right] \qquad (5)$$

$$I(t) = \left[a(T - t) + \frac{1}{2}b(T^2 - t^2) + \frac{1}{6}a\theta(T^3 - t^3) + \frac{1}{8}b\theta(T^4 - t^4) - \frac{1}{2}a\theta t^2(T - t) - \frac{1}{4}b\theta t^2(T^2 - t^2)\right]. \qquad (6)$$

(by neglecting higher powers of θ)

Putting $t = \mu_1$ in equation (4), we get

$$S_1 = (\eta - 1)(a\mu_1 + \frac{1}{2}b\mu_1^2) \qquad (7)$$

Putting $t = t_1$ in equations (5) and (6), we have

$$I(t_1) = S_1\left[1 + \theta(\mu_1 - t_1)\right] - (\eta-1)\left[a(\mu_1 - t_1) + \frac{1}{2}b(\mu_1^2 - t_1^2) + \frac{1}{2}a\theta(\mu_1^2 - t_1^2) + \frac{1}{3}b\theta(\mu_1^3 - t_1^3) - a\theta t_1(\mu_1 - t_1) - \frac{1}{2}b\theta t_1(\mu_1^2 - t_1^2)\right] \qquad (8)$$

$$I(t_1) = \left[a(T - t_1) + \frac{1}{2}b(T^2 - t_1^2) + \frac{1}{6}a\theta(T^3 - t_1^3) + \frac{1}{8}b\theta(T^4 - t_1^4) - \frac{1}{2}a\theta t_1^2(T - t_1) - \frac{1}{4}b\theta t_1^2(T^2 - t_1^2)\right]. \qquad (9)$$

So, from equations (8) and (9), we have

$$t_1 = \frac{S_1(1+\theta\mu_1) - (\eta-1)a\mu_1 - aT}{S_1\theta + \eta a\mu_1} \qquad (10)$$

From equation (10), we see that t_1 is a function of μ_1, T and S_1, so t_1 is not a decision variable.

Putting value of S_1 from equation (7) in equation (5), we have

$$
\begin{aligned}
I(t) &= (\eta - 1)(a\mu_1 + \frac{1}{2}b\mu_1^2)\left[1 + \theta(\mu_1 - t)\right] \\
&\quad - (\eta-1)\left[a(\mu_1 - t) + \frac{1}{2}b(\mu_1^2 - t^2) + \frac{1}{2}a\theta(\mu_1^2 - t^2) + \frac{1}{3}b\theta(\mu_1^3 - t^3) - a\theta t_1(\mu_1 - t) - \frac{1}{2}b\theta t_1(\mu_1^2 - t^2)\right]
\end{aligned} \quad (11)
$$

Putting $t = t_1$, in equation (5), we get

$$
\begin{aligned}
Q &= (\eta - 1)(a\mu_1 + \frac{1}{2}b\mu_1^2)\left[1 + \theta(\mu_1 - t_1)\right] \\
&\quad - (\eta-1)\left[a(\mu_1 - t_1) + \frac{1}{2}b(\mu_1^2 - t_1^2) + \frac{1}{2}a\theta(\mu_1^2 - t_1^2) + \frac{1}{3}b\theta(\mu_1^3 - t_1^3) - a\theta t(\mu_1 - t_1) - \frac{1}{2}b\theta t_1(\mu_1^2 - t_1^2)\right]
\end{aligned} \quad (12)
$$

Based on the assumptions and descriptions of the model, the total annual relevant profit (π), include the following elements:

(i) Ordering cost (OC) = A (13)

(ii) $HC = \int_0^T (x+yt)I(t)dt = \int_0^{\mu_1}(x+yt)I(t)dt + \int_{\mu_1}^{t_1}(x+yt)I(t)dt + \int_{t_1}^{T}(x+yt)I(t)dt$

$$
\begin{aligned}
&= -x\left(aT + \frac{1}{2}bT^2 + \frac{1}{6}a\theta T^3 + \frac{1}{8}b\theta T^4\right)t_1 + \frac{1}{4}y\left(\frac{1}{2}\eta b - \frac{1}{2}b\right)\mu_1^4 + \frac{1}{2}x(\eta a - a)\mu_1^2 - \frac{1}{3}\left(x\left(-\frac{1}{2}b - \frac{1}{2}a\theta T - \frac{1}{4}b\theta T^2\right) - ya\right)t_1^3 \\
&+ \frac{1}{3}\left(x\left(-\frac{1}{2}b - \frac{1}{2}a\theta T - \frac{1}{4}b\theta T^2\right) - ya\right)T^3 - x\left(\frac{1}{2}\eta a\theta\mu_1^2 + \frac{1}{6}\eta b\theta\mu_1^3 - \frac{1}{2}a\theta\mu_1^2 - \frac{1}{6}b\theta\mu_1^3\right)\mu_1 + \frac{1}{5}\left(\frac{1}{8}xb\theta + \frac{1}{3}ya\theta\right)T^5 \\
&+ \frac{1}{4}\left(\frac{1}{3}xa\theta + y\left(-\frac{1}{2}b - \frac{1}{2}a\theta T - \frac{1}{4}b\theta T^2\right)\right)T^4 + \frac{1}{2}\left(-xa + y\left(a + \frac{1}{2}bT^2 + \frac{1}{6}a\theta T^3 + \frac{1}{8}b\theta T^4\right)\right)T^4 - \frac{1}{5}\left(\frac{1}{8}xb\theta + \frac{1}{3}ya\theta\right)t_1^5 \\
&+ \frac{1}{4}\left(x\left(-\frac{1}{6}\eta b\theta + \frac{1}{6}b\theta\right) + y\left(-\frac{1}{2}b + \frac{1}{2}\eta b + \frac{1}{2}a\theta - \frac{1}{2}\eta a\theta\right)\right)t_1^4 + \frac{1}{2}\left(x(\eta a - a) + y\left(\frac{1}{2}\eta a\theta\mu_1^2 + \frac{1}{6}\eta b\theta\mu_1^3 - \frac{1}{2}a\theta\mu_1^2 - \frac{1}{6}b\theta\mu_1^3\right)\right)t_1^2 \\
&+ \frac{1}{3}\left(x\left(\frac{1}{2}\eta b - \frac{1}{2}b\right) + y(\eta a - a)\right)\mu_1^3 \\
&- \frac{1}{48}yb\theta t_1^6 - \frac{1}{4}\left(x\left(-\frac{1}{6}\eta b\theta + \frac{1}{6}b\theta\right) + y\left(-\frac{1}{2}b + \frac{1}{2}\eta b + \frac{1}{2}a\theta - \frac{1}{2}\eta a\theta\right)\right)\mu_1^4 - \frac{1}{2}\left(x(\eta a - a) + y\left(\frac{1}{2}\eta a\theta\mu_1^2 + \frac{1}{6}\eta b\theta\mu_1^3 - \frac{1}{2}a\theta\mu_1^2 - \frac{1}{6}b\theta\mu_1^3\right)\right)\mu_1^2 \\
&- \frac{1}{2}\left(x\left(-\frac{1}{2}b + \frac{1}{2}\eta b + \frac{1}{2}a\theta - \frac{1}{2}\eta a\theta\right) + y(\eta a - a)\right)\mu_1^3 + \frac{1}{5}y\left(-\frac{1}{6}\eta b\theta + \frac{1}{6}b\theta\right)t_1^5 - \frac{1}{2}\left(-xa + y\left(-\frac{1}{2}b - \frac{1}{2}a\theta T - \frac{1}{4}b\theta T^2\right)\right)t_1^4 \\
&+ x\left(\frac{1}{2}\eta a\theta\mu_1^2 + \frac{1}{6}\eta b\theta\mu_1^3 - \frac{1}{2}a\theta\mu_1^2 - \frac{1}{6}b\theta\mu_1^3\right)t_1 \\
&+ \frac{1}{3}\left(x\left(-\frac{1}{2}b + \frac{1}{2}\eta b + \frac{1}{2}a\theta - \frac{1}{2}\eta a\theta\right) + y(\eta a - a)\right)t_1^3 + x\left(aT + \frac{1}{2}bT^2 + \frac{1}{6}a\theta T^3 + \frac{1}{8}b\theta T^4\right)T - \frac{1}{5}y\left(-\frac{1}{6}\eta b\theta + \frac{1}{6}b\theta\right)\mu_1^5 + \frac{1}{48}yb\theta T^6
\end{aligned}
$$

(14)

(by neglecting higher powers of θ)

(iii) $DC = c\left(\int_{\mu_1}^{t_1}\theta I(t)dt + \int_{t_1}^{T}\theta tI(t)dt\right)$

$$
= c\theta\left(
\begin{aligned}
&\eta a\mu_1 t_1 + \frac{1}{2}\eta b\mu_1^2 t_1 - a\mu_1 t_1 - \frac{1}{2}b\mu_1^2 t_1 + \eta a\theta\mu_1\left(\mu_1 t_1 - \frac{1}{2}t_1^2\right) + \frac{1}{2}\eta b\theta\mu_1^2\left(\mu_1 t_1 - \frac{1}{2}t_1^2\right) - a\theta\mu_1\left(\mu_1 t_1 - \frac{1}{2}t_1^2\right) \\
&- \frac{1}{2}b\theta\mu_1^2\left(\mu_1 t_1 - \frac{1}{2}t_1^2\right) - \eta a\left(\mu_1 t_1 - \frac{1}{2}t_1^2\right) - \frac{1}{2}\eta b\left(\mu_1^2 t_1 - \frac{1}{3}t_1^3\right) - \frac{1}{2}\eta a\theta\left(\mu_1^2 t_1 - \frac{1}{3}t_1^3\right) - \frac{1}{3}\eta b\theta\left(\mu_1^3 t_1 - \frac{1}{4}t_1^4\right) \\
&+ \eta a\theta\left(\frac{1}{2}\mu_1 t_1^2 - \frac{1}{3}t_1^3\right) + \frac{1}{2}\eta b\theta\left(\frac{1}{2}\mu_1^2 t_1^2 - \frac{1}{4}t_1^4\right) + a\left(\mu_1 t_1 - \frac{1}{2}t_1^2\right) + \frac{1}{2}b\left(\mu_1^2 t_1 - \frac{1}{3}t_1^3\right) + \frac{1}{2}a\theta\left(\mu_1^2 t_1 - \frac{1}{3}t_1^3\right) \\
&+ \frac{1}{3}b\theta\left(\mu_1^3 t_1 - \frac{1}{4}t_1^4\right) - a\theta\left(\frac{1}{2}\mu_1 t_1^2 - \frac{1}{3}t_1^3\right) - \frac{1}{2}b\theta\left(\frac{1}{2}\mu_1^2 t_1^2 - \frac{1}{4}t_1^4\right)
\end{aligned}
\right)
$$

$$-c\theta\left(\frac{1}{2}\eta a\mu_1^2+\frac{1}{6}\eta b\mu_1^3-\frac{1}{2}a\mu_1^2-\frac{1}{6}b\mu_1^3+\frac{1}{3}\eta a\theta\mu_1^3+\frac{1}{8}\eta b\theta\mu_1^4-\frac{1}{3}a\theta\mu_1^3-\frac{1}{8}b\theta\mu_1^4\right)$$

$$+c\theta\left(\frac{1}{48}b\theta T^6+\frac{1}{15}a\theta T^5+\frac{1}{4}\left(-\frac{1}{2}b-\frac{1}{2}a\theta T-\frac{1}{4}b\theta T^2\right)T^4-\frac{1}{3}aT^3+\frac{1}{2}\left(aT+\frac{1}{2}bT^2+\frac{1}{6}a\theta T^3+\frac{1}{8}b\theta T^4\right)T^2\right) \quad (15)$$

$$-c\theta\left(\frac{1}{48}b\theta t_1^6+\frac{1}{15}a\theta t_1^5+\frac{1}{4}\left(-\frac{1}{2}b-\frac{1}{2}a\theta T-\frac{1}{4}b\theta T^2\right)t_1^4-\frac{1}{3}at_1^4+\frac{1}{2}\left(aT+\frac{1}{2}bT^2+\frac{1}{6}a\theta T^3+\frac{1}{8}b\theta T^4\right)t_1^2\right)$$

(iv) $$SR=p\left(\int_0^T(a+bt)dt\right)=p\left(aT+\frac{1}{2}bT^2\right) \quad (16)$$

The total profit (π) during a cycle, T consisted of the following:

$$\pi=\frac{1}{T}[SR-OC-HC-DC] \quad (17)$$

Substituting values from equations (13) to (16) in equation (17), we get total profit per unit. Putting $\mu_1= v_1T$ and value of t_1 from equation (10) in equation (17), we get profit in terms of T. The optimal value of T = T* (say), which maximizes profit π can be obtained by differentiating it with respect to T and equate it to zero

i.e. $$\frac{d\pi}{dT}=0 \quad (18)$$

provided it satisfies the condition

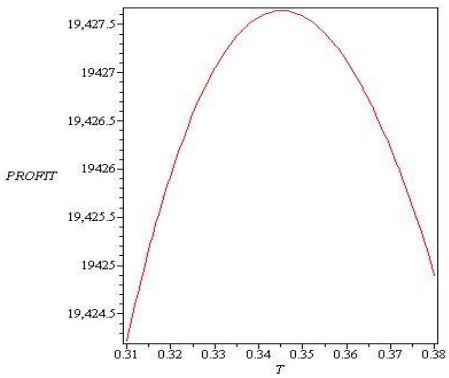
$$\frac{d^2\pi}{dT^2}<0. \quad (19)$$

IV. NUMERICAL EXAMPLE

Considering A= Rs.100, a = 500, b=0.05, η=2, c=Rs. 25, p=Rs. 40, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, in appropriate units. The optimal value T*=0.3740 and Profit*= Rs. 19472.9465 and optimum order quantity Q*=93.0725.

The second order condition given in equation (19) is also satisfied. The graphical representation of the concavity of the profit function is also given.

T and Profit



Graph 1

V. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1 Sensitivity Analysis

Parameter	%	T	Profit	Q
a	+20%	0.3422	23423.3130	102.2012
	+10%	0.3570	21447.5640	97.7319
	-10%	0.3936	17499.6354	88.1039
	-20%	0.4168	15527.8558	82.8852
θ	+20%	0.3696	19467.8390	91.9790

	+10%	0.3717	19470.3821	92.4761
	-10%	0.3762	19475.5327	93.6192
	-20%	0.3786	19478.1412	94.2156
x	+20%	0.3457	19428.0395	86.0628
	+10%	0.3590	19450.0516	89.3444
	-10%	0.3909	19496.8364	97.2470
	-20%	0.4103	19521.8587	102.0661
A	+20%	0.4085	19421.8341	101.6191
	+10%	0.3917	19446.8260	97.4458
	-10%	0.3553	19500.3698	88.3998
	-20%	0.3355	19472.9465	83.5266

From the table we observe that as parameter a increases/ decreases average total profit and optimum order quantity also increases/ decreases.

Also, we observe that with increase and decrease in the value of θ and x, there is corresponding decrease/ increase in total profit and optimum order quantity.

From the table we observe that as parameter A increases/ decreases average total profit decreases/ increases and optimum order quantity increases/ decreases.

VI. CONCLUSION

In this paper, we have developed a production inventory model for deteriorating items with linear demand with different deterioration rates. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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